Assignment 4
Math 2413

September 23, 2008

1. Read bogus §2.8-9.
2. Look at the bogus problems 2.9.1-12,20,31.
3. Read bogus §4.1
4. Look at the bogus problems 4.1.1,2,9,10,14,16,17,18.
5. Consider the admissible class $\mathcal{A} = \{u \in C^1[0,1] : u(0) = 0, u(1) = 0, \int u = 1\}$. Use this admissible class and find all minimizers of Dirichlet energy
$$\int_0^1 [u'(t)]^2 dt.$$
6. Formulate the model and derive the E-L equation for a hanging chain.
7. Read bogus §3.1.
8. Look at bogus problems 3.1.1,8,9.
9. Use the local existence/uniqueness theorem along with the growth estimate obtained in class (i.e., bogus Theorem 7.15 $|\mathbb{X}_1 - \mathbb{X}_2| \le |\mathbb{X}_1(t_0) - \mathbb{X}_2(t_0)|e^{M(|t-t_0|)}$) to show a global (or long time) existence theorem for linear equations:

Theorem: If the coefficient functions $p_1(x), \ldots, p_n(x), f(x)$ are continuous on all of $\mathbb{R}$, then the IVP

$$\begin{cases} \sum p_j y^{(j)} = f \\ y(x_0) = y_0 \\ \ldots \\ y^{(n-1)}(x_0) = y_0^{n-1} \end{cases}$$

has a C^n solution $y = y(x)$ defined for all real numbers x.

10. Vocabulary: Linear ODE, Linear independence/dependence of solutions, Constraints, Lagrange Multipliers, Phase Diagram, Stability