Assignment 3
Math 2413

August 29, 2008

1. Read bogus §2.2.
2. Look at the bogus problems 2.2.1,12,16,28,34,37,38.
3. Read Ince §3.1-3 (which corresponds to bogus §2.7).
4. Consider the admissible class $\mathcal{A} = \{u \in C^1[0,1] : u(0) = 0, u(1) = b\}$.
   (a) Use this admissible class to model paths connecting $(0,0)$ to $(1,b)$. Find the shortest path.
   (b) Use this admissible class and find all minimizers of Dirichlet energy
   $$\int_0^1 [u'(t)]^2 dt.$$
5. Let $y = \lim y_j$ be the limit solution of $y' = f(x,y)$ obtained by successive approximations. Let $\tilde{y}$ be another solution satisfying the same initial value. Prove inductively an estimate
   $$|\tilde{y}(x) - y_j(x)| \leq M_j$$
   where $M_j \to 0$. Conclude that $\tilde{y}(x) = y(x)$ for all x.
6. Go through examples 1°-5° on page 70 of Ince.
7. Characterize $\{\psi \in C_c^\infty : \psi = \eta' \text{ for some } \eta \in C_c^\infty\}$ in terms of the condition $\int \psi = 0$.
8. Vocabulary: Existence/Uniqueness, Lipschitz, Cauchy (sequence), uniform (convergence), induction, linear (ODEs), $L^1[a,b]$.