# Assignment 1
# Math 2413

August 18, 2008

1. Read bogus §1.2-3 and §2.1. §1.1 is (mostly) nonsense.

2. Look at the bogus problems 1.1.1-11; 1.2.8,10,12,13,16; 1.3.5,8,12,14,23,26,27

3. Show that if $x = x(t)$ is a continuously differentiable function with $x(a) = 0$ and $x' = x$ in some open interval $I$ containing $t = a$, then $x(t) \equiv 0$ on the interval $I$.

   Notes:

   (a) An *open interval* is one of the form $(b, c) = \{t : b < t < c\}$.

   (b) *Continuously differentiable* means that the function has a well defined derivative and that the derivative is continuous. The set of continuously differentiable real valued functions on an interval $I$ is denoted by $C^1(I)$. The set of continuous functions on $I$ is denoted by $C^0(I)$. If the context makes the domain (interval) of the functions clear, then we can just write $C^0$ and $C^1$.

4. Solve the IVP

$$\begin{cases} y'' = -9.8, \\ y(0) = h_0, \\ y'(0) = v_0. \end{cases}$$

5. A first order system equivalent to the equation $y'' = -9.8$ is given by

$$\begin{cases} y' = v, \\ v' = -9.8. \end{cases}$$

In view of the previous problem, what would the appropriate initial condition look like for this system? Explain why we call this data an "initial position" instead of an "initial position and velocity" as would be appropriate for Newton's Law.

6. Let $f_0$ and $\eta$ be a fixed functions and assume $\mathfrak{F}[f_0] \leq \mathfrak{F}[f_0 + \epsilon\eta]$ for all $\epsilon$.

   (a) Draw a picture of what the graph of $g(\epsilon) = \mathfrak{F}[f_0 + \epsilon\eta]$ might look like near $\epsilon = 0$.

   (b) Assume that $g$ is differentiable, and use one dimensional calculus on $g(\epsilon)$ to conclude that

   $$\frac{d}{d\epsilon}\mathfrak{F}[f_0 + \epsilon\eta]\Big|_{\epsilon=0} = 0.$$

   Note: The *graph* of a (real valued) function $g$ defined on an interval $I$ is the set of pairs in the plane defined by

   $$\{(t, g(t)) : t \in I\}.$$

7. Vocabulary: ODE, regular (equation), normal (form), independent (variable—review), argument (review), dependent variable (review), right side (of equation), FTC (equations), IVP, autonomous, explicit solution, numerical solution (lecture 2), linear equations (later), modeling, dynamical systems (later), bifurcation (later), system of ODEs, calculus of variations (will understand more fully later), optimization, minimization, maximization, minimizer, maximizer, graph (of a function—review), parameterization (review), scaling factor (review), variation, vector space (review), dimension (review), admissible (functions or class), necessary condition (review).